EMERGENCE OF QUANTUM MECHANICS

Newtonian Mechanics was successful in explaining motion of heavenly bodies and in the motion of terrestrial objects. But when these concepts were applied to particles of small dimensions like atom, electrons, protons, they failed to explain observed features. Thus classical mechanics was unable to explain many phenomena observed by experimental physicist.

1) Black body Radiation

It is well known that every object absorbs and emits energies incident on them. A perfect blackbody is one which emits all its energy given it in terms of heat. The radiation emitted by a perfect black body is often referred to as black body radiation.

If $E_\lambda d\lambda$ represents the energy radiated per unit area per second for wave lengths in the lange $\lambda \text{ and } \lambda + d\lambda,$ then the emission curve of a black body is as shown

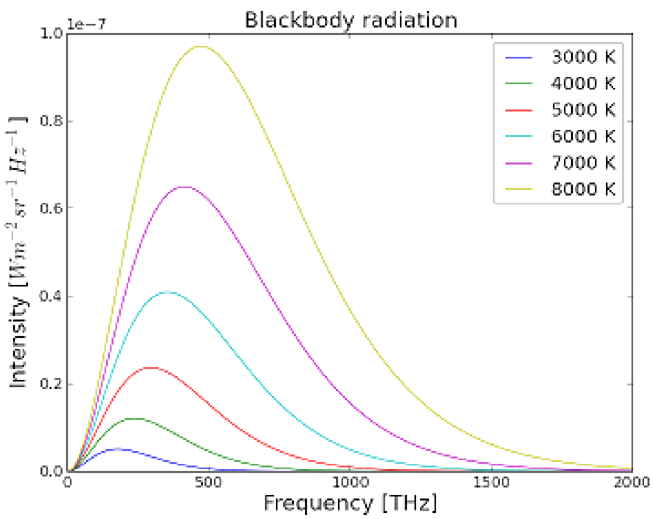

Fig-1

The characteristics of the curve are

1. For every wave length the emissive power E_λ increase with increase of temperature.

2. At a constant temperature E_λ increases till it becomes a maximum (λ_m) at a specific wavelength and E_λ decreases as λ is increased further.
3. The maximum emission happens at a wavelength λ_m for a particular temperature. This λ_m shifts towards shorter wavelength as Temperature is increased. The relation ship between λ_m and temperature T is found to be
   $\lambda_m T = a\ constant$
   Where T is the absolute temperature of the emitter. This relation is known as wien's displacement law.
4. The emissive power maximum E_m and the absolute temperature of the emitter is given by
   $$\frac{Em}{T} = a\ constant$$
5. The area under the curve at a particular temperature represents the total radiation emitted per unit area per unit time over all wavelengths ie
   $$E = \int_0^\infty E_\lambda d\lambda = \sigma T^4$$
   Where σ is the stefan's constant equal to 5.6697 × 10^{-8} W/m^2k^4. The above equation is called Stefan's law. Many scientists attempted to explain the observed spectral distribution of energy spectrum of black body radiation on the basis of classical Mechanics and electromagnetic theory. But this explanation attained only partial success. In 1901, Max planck proposed quantum theory of radiation and gave a satisfactory explanation for the black body spectrum.

Wien's Theory

In 1893, Wien proposed a relation ship between $E_\lambda\ and\ T$ empirically as

$$E_\lambda (T) d\lambda = \frac{A}{\lambda^5} f(\lambda T) d\lambda$$

Where A is a constant and $f(\lambda T)$ is a function of λT putting $x = \lambda T$

$$E = \int_0^\infty E\lambda d\lambda = A \int_0^\infty \frac{f(\lambda T)}{\lambda^5} d\lambda$$

$\lambda \frac{x}{T}$ $$E = A\int_0^\infty \frac{f(\lambda T)}{\left(\frac{x}{T}\right)^5}.\frac{d(\lambda T)}{T}$$

$$E = AT^4\int_0^\infty \frac{f(x)}{x^5}dx$$

$\int_0^\infty \frac{f(x)}{x^5}dx$ is a definite integral and hence a constant

$\therefore\ E = \sigma T^4$

This is Stefan – Boltzmann law

Differentiating above expression

$$\frac{dE_\lambda}{d\lambda} = \frac{-5A}{\lambda^6} f(\lambda T) + \frac{AT}{\lambda^5} f'(\lambda T)$$

$E_\lambda = E_m$ at $\lambda = \lambda_m$

Then $\left(\frac{dE_\lambda}{d\lambda}\right)_{max} = 0$

or $Tf'(\lambda_m T) = \frac{5f(\lambda_m T)}{\lambda_m}$

ie $\lambda m f'(x_m) = 5f(x_m) = 0$

where $x_m = \lambda_m T$

The above equation can have only one solution.

$x_m = \lambda m T = a\ constant$

This is found to be true in blackbody emissions and is known as Wien's displacement law. This predicts the shift of emission peak towards lower λ with increase in T.

This law holds good for low values of λT but for higher values, the predicted value of E_λ is lower than that of experimentally obtained value.

Applying laws of equipartition of energy to the electro magnetic vibrations Rayleigh and Jeen obtained a formula for energy density inside an enclosure with perfectly reflecting walls. The energy density in the frequency range at temperature T is given as

$$U_r d_z = \frac{8\pi v^2 kT}{c^3} dz$$

Where U_2 = is the energy per unit volume per unit frequency range at V, k, the Boltzmann constant

$$V = \frac{C}{\lambda} \quad d_z = \frac{-C}{\lambda^2} d\lambda$$

Hence $$U_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda$$

The above equation is called Rayleigh – Jeen's law. This equation predicts the spectral range at lower wavelength out fails in the shorter wavelength side. Also as $\lambda \rightarrow 0 \ U_\lambda \rightarrow \infty$ for constant temperature. This is not experimentally correct.

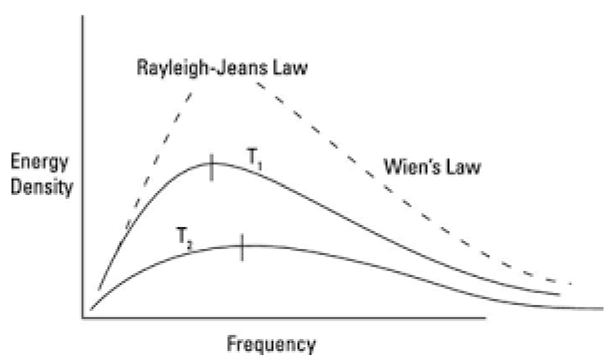

Fig-2

In 1901 Max Planck put forward new postulate regarding the nature of vibration of a linear harmonic oscillator. He introduced two ideas.

1. An oscillator has a discrete set of energies which are integral multiples of a finite quantum of energy $E = h\upsilon$ where h is a constant called Planck's constant.

   $\therefore E_n = nh\upsilon$

   Where n = 0, 1, 2, 3, ……

2. As long as the oscillator possesses energy $E = nh\upsilon$ it can not emit or absorb energy. Emission or absorption occurs when the oscillator jumps from one energy state to another.

   ie $\Delta E = E_2 - E_1 = (n_2 - n_1) h\upsilon$

$\Delta E = h\upsilon$ it $n_2 = n_1 + 1$

The estimated value of h is found to be 6.6 × 10^{-34} JS

Consider an enclosure with N oscillators having total energy E then average energy per oscillator is

$$(E) = \frac{E}{N}$$

If there are No oscillators with energy E = 0 and N_1 with energy $E = h\upsilon,\ N_2$ with energy $E = 2h\upsilon$ etc.

Then $N = N_0 + N_1 + N_2 + \ldots..$

$$E = O + h\upsilon N_1 + 2h\upsilon N_2 + \ldots..$$

But from Maxwell Boltzmann law

$$N_1 = N_0 e^{-E/kT}$$

and $N_2 = N_0 e^{-E/kT}$

$$\therefore\ N = N_0 + N_0 \exp\left(-\frac{h\upsilon}{kT}\right) + N_0 ex\left(\frac{-2h\upsilon}{kT}\right) + \ldots..$$

$$= N_0\left[1 + \exp\left(\frac{-h\upsilon}{kT}\right) + \exp\left(\frac{-2h\upsilon}{kT}\right) + \ldots..\right]$$

$$E = N_0 E\left[\exp\left(\frac{-E}{kT}\right) + \exp\frac{-2E}{kT} + \exp\frac{-3E}{kT} + \ldots..\right]$$

Where $E = h\upsilon$

Put $y = \exp\left(\frac{-E}{kT}\right)$ then

$$N = N_0[1 + y + y^2 + \ldots..] = \frac{N_0}{1-y}$$

$$E = N_0 \in (y + y^2 + y^3 + \ldots..] = N_o \frac{¿y}{(1-y)^2}$$

$$¿E> = \frac{N_0 \in \frac{y}{(1-y)^2}}{\frac{N_0}{1-y}} = \frac{h\upsilon \exp\left(\frac{-E}{kT}\right)}{1 - \exp\left(^{-E}/_{kT}\right)}$$

$$¿E> = \frac{h\upsilon}{\exp\frac{h\upsilon}{lT} - 1}$$

If $h\upsilon << kT \quad \exp\left(\frac{h\upsilon}{kT}\right) = 1 + \frac{h\upsilon}{kT}$

$$¿E>= \frac{h\upsilon}{1+\frac{h\upsilon}{kT}-1} \approx kT$$

Under $h\upsilon << kT$ ¿ $<E>$¿ reduces to classical equation.

If can be show that no. of oscillators per unit volume

$$N(\upsilon)dv = \frac{8\pi v^2}{c^3}dv$$

∴ Energy/unit volume

$$U_v dv = N(v)dv \times ¿E>¿$$

$$= \frac{8\pi v^2}{c^3} \frac{h\upsilon}{\exp\frac{h\upsilon}{kT}-1}$$

$$U_v d_v = \frac{8\pi h v^3}{c^3} \frac{1}{[\exp\frac{hv}{kT}-1]}$$

This is known as Planck's radiation law.

Photoelectric effect

The phenomenon in which electrons are emitted from a metal surface when irradiated with light is called photo electric effect (PEE). There metals are called photo sensitive materials and the ejected electrons are called photo electrons. Some metals exhibiting photo sensitivity are Na, K, Ce etc.

This effect was first observed by Henrich Hertz in 1887 and was studied in detail by Lennard.

Lennard Expt

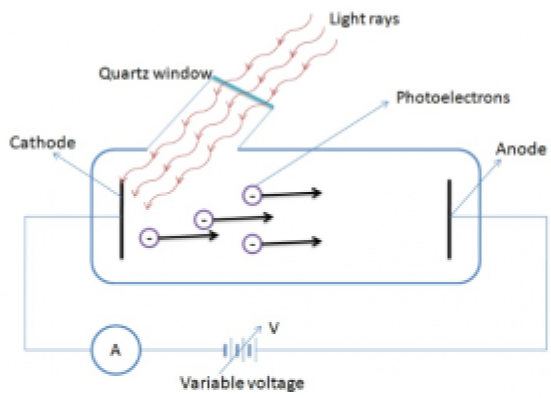

Fig-3

Two metal plates C and D are mounted on an evaluated tube. C is coated with a photo sensitive materials and is applied a negative potential. The plate A is kept at zero potential. When radiation is incident on C electrons are ejected out. These are attracted to A and is collected at D. The current is detected by electrometer E. The entire tube is placed in a mag. Field B which is perpendicular to the plane of the paper.

The path of electrons is shifted and are collected at E.

If V is the potential difference then

KE of electrons $=\frac{1}{2}mv^2=eV$ …..(1)

When mag. Fields is applied

$$BeV=\frac{mV^2}{R}$$ …..(2)

From (1) and (2) $e/m=\frac{2V}{B^2R^2}$

Where R is the radius of curvature of e- track B the magnetic field.

The observed e/m matched with that of electrons.

If a +ve potential is applied to C, the current is found to decrease and ultimately ejection is stopped. This potential is called stopping potential. Then

$$\frac{1}{2}mV_m^2=eV_s$$

Some observations

1. At stopping potential, no current is found when intensity of incident light is increased.
2. If C is applied –ve potential, with intensity of incident light the current increases.
3. For a constant intensity, it incident frequency is increase stopping potential also increases.
4. The photo current is found to be independent of frequency of incident light and depends only on intensity.
5. When the potential is increased then the photo current is found to increase linearly No. of photo electrons emitted per second by a surface is directly proportional to intensity of light.

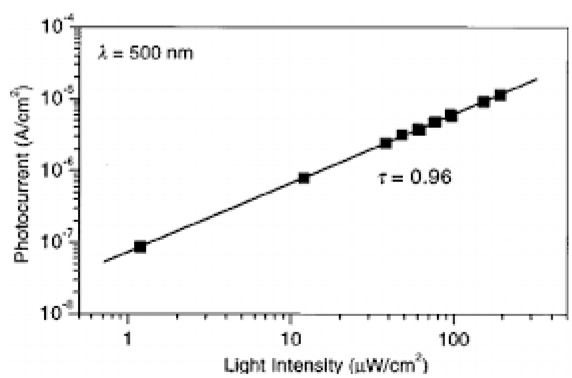

Fig-4

6. If C is maintained a +ve potential with respect to A then the potential is called retarding potential. As this potential is increased the photo current reduces and became zero for a particular potential called stopping potential.
7. It is observed that stopping potential increases with increase of incident frequency.

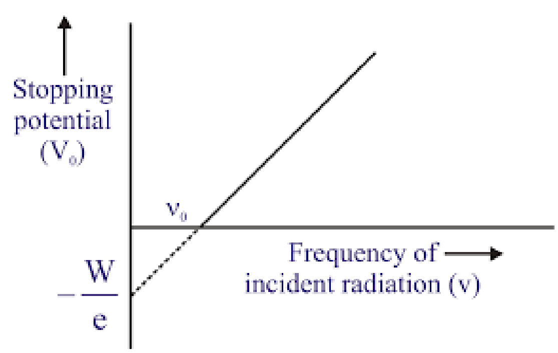

Fig-5

8. It is observed that there exists a minimum frequency above which only photo emission starts flowing is a characteristic for a material. This minimum frequency is called threshold frequency (ν_0)

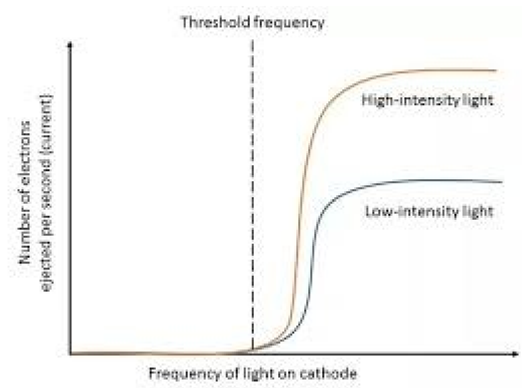

Fig-6

9. Photo electric effect happens instantaneously. Ie there is no time lag between illumination with light and ejection of electrons.

Failure of Electromagnetic Theory

According to nave theory, PEE should occur for any frequency, if intensity is enough to eject an electron. But experimentally there is a threshold frequency for every metal.

As per wave theory, highly intense incident radiation can impart more energy to electron. But experimentally energy increases with frequency and not intensity of incident radiation.

According to wave theory, energy is distributed in the wive front. So any electron on the surface absorbs only little energy. Hence time is required to eject electron from surface. But experimentally there is no time lag between falling of radiation and ejection of electron.

Einstein's Photo electric equation

According to Max Planck energy emission or absorption occur discretely as quanta as

$E = nh\nu$ n = 1, 2, 3….

Einstein extended Planck's idea. He proposed that light is not only emitted as quanta, it travels and hits an electron as quanta. This quantum transfers entire energy when it hits an electron on the surface of a metal with out any time lag. Hence a minimum amount of energy is needed to release the electron from the surface.

Photo electric equation

When a radiation of energy $h\nu$ is incident on he surface of a metal, an energy of $h\upsilon_0$ is used to knock the electron from the metal to the surface. This energy is called work function φ_0. This is different for different metals. Thus emission of electron is possible only if $h\upsilon > \varphi_0$. Thus there exists a minimum frequency called threshold frequency υ_0 for PEE to occur.

The balance of energy ie $h\upsilon - \varphi_0$ is utilized to impart kinetic energy to the electron and electron comes out with a speed μ . Hence

$$h\nu = \varphi_0 = \frac{1}{2} mV^2$$

and $\varphi_0 = h\upsilon_0$ υ_0 is called threshold frequency

ie $$h\upsilon - h\upsilon_0 = \frac{1}{2} mV^2$$

The above equation is called Einstein's photoelectric equation.

1. If $\upsilon < \upsilon_0$ $\frac{1}{2} mV^2$ is –ve. That is no electron is emitted. Ie PEE takes place only if incident frequency exceeds threshold frequency. This explains experimentally observed minimum frequency.
2. The kinetic energy depends only an frequency not on intensity of incident radiation. The kinetic energy varies linearly with incident frequency which is experimentally proved.

3. An increase of intensity of incident radiation only increases the no. of electrons ejected from the surface and hence increases only current. The kinetic energy of individual electrons remains the same.
4. If $\upsilon > \upsilon_0$, the incident photon is absorbed instantaneously.

Hence Einstein's quantum theory explains correctly all experimental observations pertaining to PEE.

Application

1. In photocells
2. Photomultiplier tube
3. High dependent resistance
4. In scintillation counter
5. In solar cells.

Compton Effect

When a beam of monochromatic X-rays of wavelength λ is scattered by light elements like carbon, it is observed that the scattered X-rays have maximum intensities at two wavelengths λ and λ' and $\lambda' > \lambda$. λ corresponds to unmodified radiation while λ' corresponds to modified radiation.

This phenomenon of shift of radiation after scattering is known as Compton effect. (AH Compton of USA in 1922). The difference in wavelength $\Delta\lambda = \lambda' - \lambda$ is called Compton shift. $\Delta\lambda$ depends on angle of scattering but is independent of wavelength of incident radiation and on the nature of the scatterer.

Experiment

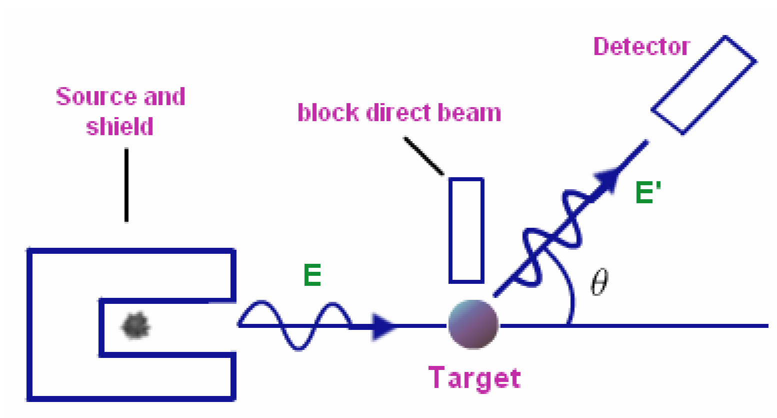

Fig-7

A beam of collimated X rays tall on a graphite block and the intensity of scattered X-rays we measured as a function of wavelength of angle of scattering. For a specific angle of scattering two peals appear one at incident wavelength λ and another at longer modified wavelength λ' .

Failure of Classical Theory

As per classical theory, X-rays are electromagnetic waves. When it is incident on a material, the electrons in the material will be forced to oscillate with same frequency υ. The oscillating electron will reradiate in same frequency υ. Hence scattered radiation should contain only one frequency υ. Also electron should radiate in all possible direction and wavelength should not vary with scattering angle.

But Compton experiment in contrary to the above theory. The wavelength gets modified and wavelength scattered is a function of scattering angle.

Compton's Theory

Compton considered light to be consisting of discrete quanta called photons and also they carry momentum.

Assumptions

1. The beam of incident radiation with frequency υ contains a stream of photons each having energy $E=h\upsilon$ traveling in the direction of beam. Each photon has a momentum $P=\frac{E}{C}=\frac{h\upsilon}{C}$
2. The scattering is due to elastic collision between photon and free electron at rest obeying law of conservation of energy and momentum. Consider a beam of monochromatic X-rays of frequency υ and wavelength λ incident on a target electron as shown in figure.

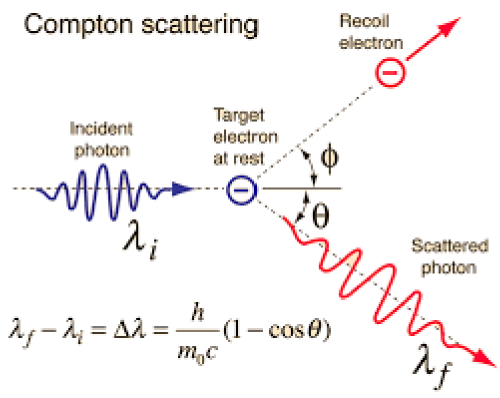

Fig-8

Let an incident photon suffer an elastic collision with free electron and gets scattered through an angle φ and electron recoils in a direction θ with direction of incidence. If the frequency of scattered photon is $\upsilon'(r'<\upsilon)$. The momenta of these photons are $\frac{h\upsilon}{C}$ *and* $\frac{h\upsilon'}{C}$

Kinetic energy of electron $=mc^2-m_0c^2$

$$E_k=\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}-m_0c^2$$

Let $\frac{v^0}{c}=\varphi$ and $mv=\frac{mv}{c}\times c=m\beta c$

Ie

$$h\upsilon=h\upsilon'+E_k$$

$$=h\upsilon'+\frac{m_0c^2}{\sqrt{1-\beta^2}}-m_0c^2$$

Applying law of conservation of linear momentum.

$$\frac{h\upsilon}{c}=\frac{h\upsilon'}{c}\cos\varphi+mv\ \cos\theta$$

$$\frac{h\upsilon}{c}=\frac{h\upsilon'}{c}\cos\varphi+\frac{m_0\beta c}{\sqrt{1-\beta^2}}\cos\theta \qquad \text{...........(1)}$$

And

$$O=\frac{h\upsilon'}{c}\sin\varphi-\frac{m_0\beta c}{\sqrt{1-\beta^2}}\sin\theta \qquad \text{…………(2)}$$

$$(1)\Rightarrow\frac{m_0\beta c}{\sqrt{1-\beta^2}}\cos\theta=\frac{h\upsilon}{c}-\frac{h\upsilon'}{c}\cos\varphi$$

$$(2)\Rightarrow\frac{m_0\beta c}{\sqrt{1-\beta^2}}\sin\theta=\frac{h\upsilon'}{c}\sin\varphi$$

Squaring and adding

$$\frac{m_0^2\beta^2c^2}{1-\beta^2}[\cos^2\theta+\sin^2\theta]=\frac{h^2\upsilon^2}{c^2}+\frac{h^2\upsilon^{12}}{c^2}-\frac{2h^2\upsilon\upsilon'}{c^2}\cos\varphi \qquad \text{…………(3)}$$

But

$$h\upsilon=h\upsilon'+\frac{m_0c^2}{\sqrt{1-\beta^2}}-m_oc^2$$

$$\frac{m_0c^2}{\sqrt{1-\beta^2}}=h\upsilon-h\upsilon'+m_0c^2$$

Squaring

$$\left[\frac{m_0c^2}{\sqrt{1-\beta^2}}\right]^2=[h\upsilon-h\upsilon'+m_0c^2]^2$$

$$[h(\upsilon-\upsilon')+m_0c^2]^2$$

$$=h^2(\upsilon-\upsilon^1)^2+m_0^2c^4+2h(\upsilon-\upsilon^1)m_0c^2$$

$$=h^2\upsilon^2+h^2\upsilon^{12}-2h^2\upsilon\upsilon'+m_0^2c^4+2hm_0c^2(\upsilon-\upsilon^1)$$

¿ both sides by c^2

$$\frac{m_0^2c^2}{1-\beta^2}-m_0^2c^2=\frac{h^2\upsilon^2}{c^2}+\frac{h^2\upsilon^{12}}{c^2}-\frac{2h^2\upsilon\upsilon^1}{c^2}+2m_0h(\upsilon-\upsilon^1)$$

……………(4)

(4) – (3)

$$O=2m_0h(\upsilon-\upsilon^1)-\frac{2h^2\upsilon\upsilon^1}{c^2}(1-\cos\varphi)$$

or $$\frac{\upsilon-\upsilon^1}{\upsilon\upsilon^1}=\frac{h}{m_{0c}}(1-\cos\varphi)$$

or $\frac{c}{\upsilon^1} - \frac{c}{\upsilon} = \frac{h}{m_0 c}(1-\cos\varphi)$

$$\lambda^1 - \lambda = \frac{h}{m_0 c}(1-\cos\varphi)$$

or $\Delta\lambda = \frac{h}{m_0 c}(1-\cos\varphi)$

$$\Delta\lambda = \lambda_c(1-\cos\varphi)$$

Where $\Delta\lambda = \lambda^1 - \lambda$, the Compton shift and $\lambda_c = \frac{h}{m_0 c}$, the Compton wavelength

$\Delta\lambda$ is independent of nature of scatterer and also on incident wavelength λ. It depends on angle of scattering.

$h = 6.6\times10^{-34} Js$ $m_0 = 9.1\times10^{-31} kg$ $c = 3\times10^8 m/s$

$$\lambda_c = \frac{h}{m_0 c} = \frac{6.6\times10^{-34}}{9.1\times10^{-31}\times3\times10^8} = 0.02434°$$

ie $\Delta\lambda = 0.0243(1-\cos\varphi)$

Thus Compton effect is a quantum phenomenon and can only the explained using quantum hypothesis of Max Planck.

Limitations of Compton theory

1. Although Compton explained the modified wave length λ^1, the presence of incident wavelength is unmodified wavelength λ could not be explained.
   This can be explained as follows the X-ray photons collide with
   a) loosely bound outer electrons and
   b) tightly bound inner electrons.
   When it incident on tightly bound e- it is not detached from atom and entire atom has to recoil. In such cases instead of man of e- the man of the atom has to be substituted in Compton formula. Since man of atom is considerably high $\Delta\lambda \rightarrow 0$ and hence unmodified wavelength also will be present.
2. Experimentally it is found that intensity of modified X-rays are greater than that of unmodified X-rays for lighter elements and reverse for heavier elements. Compton's theory could not explain this experimental fact.

Wave properties of Matter

Light shows well known effects like interference, diffraction, polarization etc which could only be explained on the basis of wave nature while some phenomena like blackbody radiation, photo electric effect, Compton effect, Raman effect etc. Could only be explained based on particle nature of light. This prompted Einstein to introduce the principle of duality. Thus it was proposed that light has dual nature.

In 1924, French theoretical physicist proposed that matter also exhibits dual nature.

De-Broglie concept

1. Nature loses symmetry :- Matter and energy are two manifestations of nature. Since radiation posses dual nature then matter also should possess such a dual native.

Wave properties of matter (contd)

According to planck – Einstein's equation

$E = h\upsilon$ where h is the Planck's constant.

The total relativistic energy

$E^2 = c^2 p^2 + m_0^2 c^4$

For a photon $m_0 = 0$

ie $E^2 = c^2 p^2$ or $E = C.P$

ie $P = \frac{E}{C}$

Now $PC = h\upsilon = \frac{hc}{\lambda}$

or $\lambda = \frac{h}{p}$

Here λ is the wavelength of wave associated with photon and p is its momentum regarding it as a particle.

Characteristics of de-Broglie waves

1. Lighter the particle greater is the de-Broglie wave length $\lambda \alpha \frac{1}{m}$
2. Smaller the velocity of the particle greater is the de-Broglie wavelength

   ie $\lambda \alpha \frac{1}{\mu}$
3. $\lambda = \infty$ when $\mu \rightarrow 0$ ie waves are provided due to motion of the body. The waves are charge independent.
4. Velocity of matter wave is not constant but depends on velocity of body. Electro magnetic waves have constant velocity.
5. Both wave and particle nature of a moving body cannot appear simultaneously in an expt.

Bohr Theory of atom

In 1913 Niels Bohr applied planck's quantum hypothesis to atom model and successfully explained origin of spectral lines in hydrogen atom.

He proposed two postulates.

1. In hydrogen atom e- revolves round in circular orbits around the nucleus such that the angular momentum is an integral multiple of $\frac{h}{2\pi}$. ie

   $$m\mu r = \frac{nh}{2\pi}$$

   Where r is the radius of the orbit, μ the velocity of electron of man m and n is called principal quantum number which has restricted values 1, 2, 3….. such orbits are called stationary orbits because electron do not radiate so long it is in this orbit.
2. Electron can jump from one orbit to another by absorbing or emitting energy. If Ei is the energy of initial orbit and Ef that in final orbit then

   $$E_i - E_f = h\upsilon$$

   Where υ is the frequency of radiation absorbed or emitted. This is known as Bohr frequency condition.

According to wave mechanics, an electron moving in a circular orbit of radius r with constant speed υ is associated with a de Broglie wavelength λ given by

$$\lambda=\frac{h}{p}=\frac{h}{m\upsilon}$$

As per Bohr postulate

$$m\upsilon r=\frac{nh}{2}\ \ ie\ 2\pi r=\frac{nh}{m\upsilon}=\frac{nh}{p}=n\lambda$$

Thus the circumference of the orbit should be an integral multiple of λ.

The waves will be in phase and such waves interface to form a stationary waves.

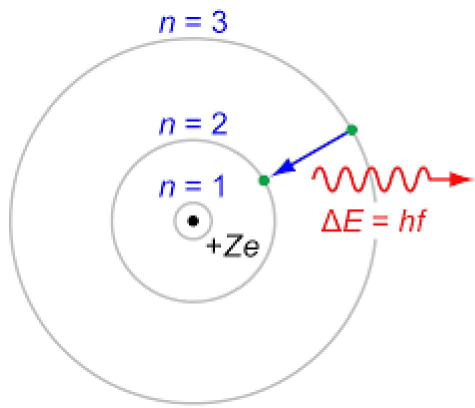

Fig-9

Energy Levels

Consider the case of hydrogen atom, where the electron revolves in a circular orbit of radius r with nucleus of charge ze =+e (z = 1) with speed υ. The centripetal free should balance electrostatic attraction ie

$$\frac{mV^2}{r}=\frac{e^2}{4\pi\in_0 r^2} \quad ....(1)$$

Also

$$m\upsilon r=n\hbar\ .....(2)\ \hbar=\frac{h}{2\pi}$$

$$\mu=\frac{n\hbar}{mr} \quad ....(3)$$

From (1)

$$r=\frac{e^2}{4\pi\in_0 mV^2}=\frac{e^2}{4\pi\in_0 .m.\frac{n^2\hbar^2}{m^2r^2}}$$

ie $r=\frac{4\pi\in_0 n^2\hbar^2}{me^2}$ ....(4)

$\therefore\ \mu=\frac{e^2}{4\pi\in_0 n\hbar}$ ....(5)

Total energy E = KE + PE

$$E=\frac{1}{2}mv^2+\frac{-e^2}{4\pi\in_0 r}=\frac{e^2}{2\times 4\pi\in_0 r}-\frac{e^2}{4\pi\in_0 r}$$

$E=-\frac{1}{2}\frac{e^2}{4\pi\in_0 r}$ ....(6)

Energy E is negative which means that electron is band to the nucleus and to separate then needs energy.

From (4)

$$E=-\frac{1}{2}\ \frac{e^2}{4\pi\in_0 \text{¿}}\cdot\frac{me^2}{4\pi\in_0 n^2\hbar^2}$$

¿

$$\therefore\ E=-\frac{me^4}{(4\pi\in_0)^2 2\hbar^2}\cdot\left[\frac{1}{n^2}\right]$$

Substituting $\text{¿}_0=8.85\times10^{-12}F/m$

$h=6.6\times10^{-34}Js$ $m=9.1\times10^{-31}kg$ $e=1.6\times10^{-19}c$

We get

$$E=-\frac{2.18\times10^{-18}}{n^2}J$$

But $1.6\times10^{-19}J=1eV$

$$\therefore\ E_n=\frac{-13.6}{n^2}eV$$

$$\frac{n=1}{E_1}=\frac{-mc^2}{2}\left[\frac{e^2}{4\pi\in_0\hbar c}\right]^2=\frac{-mc^2}{2}\alpha^2=13.6\ eV$$

$\alpha=\frac{e^2}{4\pi\in_0\hbar c}\simeq\frac{1}{137}$ is called fine structure contant

$$r_1=a_0=\hbar^2 4\pi\in_0\frac{\text{¿}}{mc^2}=\frac{\hbar^2}{mc}\cdot\frac{4\pi\in_0\hbar c}{e^2}$$

¿

$$a_0 = \frac{\hbar^2}{mc}\left(\frac{1}{\alpha}\right)$$

ie $a_0 = 0.53 \times 10^{-10} m = 0.53 A^\circ$

$$r_n = 0.53 A^\circ \times n^2$$

Bohr theory successfully explained the experimentally observed discrete spectra. According to Bohr postulate

$$\frac{E_i - E_f}{n} = \mu = \frac{mc^2\alpha^2}{2h}\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$\upsilon = \frac{C}{\lambda}$ and $\frac{1}{\lambda} = \bar{\upsilon},$ the wave number ie no. of waves per unit length

$$\therefore \frac{1}{\lambda} = \frac{mc\alpha^2}{4\pi\hbar}\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

or $\bar{\upsilon} = R_H\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$

$$R_H = \frac{mc\alpha^2}{4\pi\hbar} = \frac{mc}{4\pi\hbar}\left[\frac{e^2}{4\pi\in_0\hbar c}\right]^2$$

or $R_H = \frac{me^4}{(4\pi\hbar)^2\in_0^2 C}$

R_H is called Rhyberg constant given by $R_H = 1.097 \times 10^7 \; m^{-1}$

This value agrees well with that with experimental results.

$$\upsilon = R_H\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

1. Lyman series

   If nf =1 ni = 2, 3, 4, …….

$$\bar{\upsilon}_{LI} = R_H\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3}{4}R_H \quad \lambda_{LI} = \frac{4}{3R_H}$$

$$\bar{\upsilon}_{LII} = R_H\left[\frac{1}{1^2} - \frac{1}{3^2}\right] = \frac{8}{9}R_H \quad \lambda_{LII} = \frac{9}{8R_H}$$

$$\bar{\upsilon}_{LIII} = R_H\left[\frac{1}{1^2} - \frac{1}{4^2}\right] = \frac{15}{16}R_H \quad \lambda_{LIII} = \frac{16}{16R_H}$$

2. Balmer series

If $n_f = 2$ $\quad n_i = 3, 4, 5.....$ $\quad$ the spectral series is called Balmer series.

$$\bar{\upsilon}_{BI} = R_H\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5}{36} R_H \quad \lambda_{BI} = \frac{36}{5 R_H}$$

$$\bar{\upsilon}_{BII} = R_H\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{12}{16} R_H \quad \lambda_{BII} = \frac{4}{3 R_H}$$

$$\bar{\upsilon}_{BIII} = R_H\left[\frac{1}{2^2} - \frac{1}{5^2}\right] = \frac{21}{50} R_H \quad \lambda_{BIII} = \frac{50}{21 R_H}$$

This series came in the visible region.

If $n_f = 3$ $\; n_i = 4, 5.........$, the paschen series appear is the infrared region.

Energy level diagram

The quantized energy values can be found by

$$E_n = \frac{-13.6}{n^2} eV$$

Where n can take value 1, 2, 3……..

Accordingly the energy values allowed are -13.6 eV, -3.4 eV, -1.5 eV, -0.8eV and so on

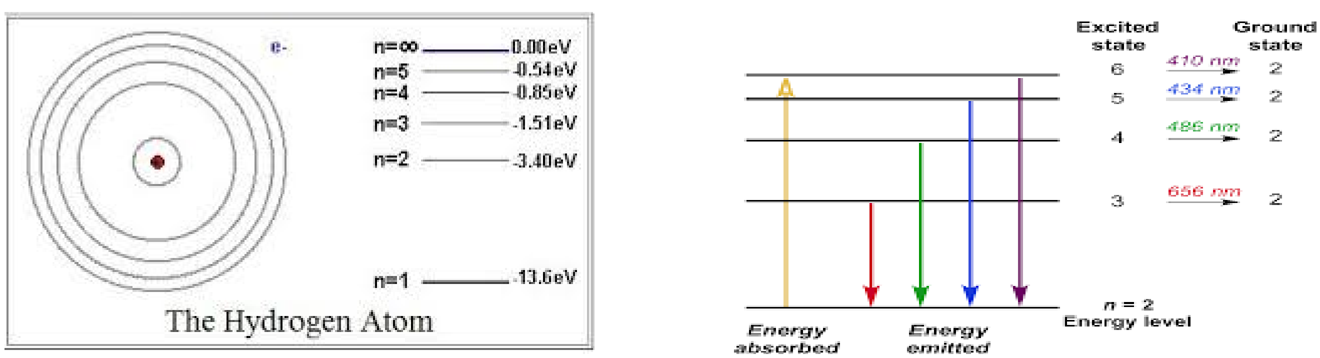

Fig-10

N = 1 level is called ground state where the atom is in the normal state. As n increases the energy also increases and they are called excited states. For

$n \rightarrow \infty \quad E_n \rightarrow 0$ is the electron is removed from the atom. If E > 0 the energy level are +ve which form the continuum.

Advantages of Bohr theory

1. It provides explanation for origin of spectral lines.
2. The experimental value of Rhydberg constant R_H agrees wall with that calculated using Bohr – Theory.
3. The value of e/m calculated using Bohr theory agrees well with that of experiment.
4. The value of Bohr radius a_0 = 0.528 A° agrees well with the value obtained using other methods.

Failures of Bohr Theory

1. The theory could not produce the distribution of e- with in the atom.
2. In this theory the equilibrium of e- is explained using classical laws while emission of radiation is explained using quantum rules.
3. If could not explain the fine structure of $H_\alpha, H_\beta, H_r \text{ and } H_\delta$ lines.
4. It could not give any idea of intensity of spectral lines.
5. The selection rules cannot be calculated using this theory.
6. The e- orbits were taken to he circular while they can be elliptical also.

Bohr's Correspondence Principle

The classical physics successfully explained the dynamics of microscopic bodies. According to Bohr theory for higher values of quantum number n, the successive difference between energy levels decreases and the energy spectrum levels continuous. The Bohr's correspondence principle states that "the Behaviour of an atomic system as predicted by quantum theory tends asymptotically to that expected in classical theory in transitions involving states of large quantum numbers.

$$\upsilon_{classical} = \frac{1}{T} = \frac{1}{\left[\frac{2\pi r}{\mu}\right]} = \frac{\mu}{2\pi r}$$

ie $\upsilon_{classical}=\frac{me^4}{2\pi n^3 h^3}$

Since $\upsilon=\frac{e^4}{4\pi\in_0 n\hbar}$ $\qquad r=\frac{4\pi\in_0 n^2\hbar^2}{me^2}$

In quantum Mechanics

$$\upsilon_q=\frac{E_f-E_i}{2\pi\hbar}=\frac{me^4}{4\pi\hbar^3}\left[\frac{1}{n_i^2}-\frac{1}{n_f^2}\right]$$

If $n_i=n \;\; n_f=n+1$

$$\upsilon_a=\frac{me^4}{4\pi\hbar 3}\left[\frac{1}{n^2}-\frac{1}{(n+1)^2}\right]$$

ie $\upsilon_a=\frac{me^4}{2\pi\hbar 3}\,\frac{2n+1}{n^2(n+1)^2}$ $\qquad n\rightarrow\infty \qquad n+1\approx n \qquad zn+1\approx 2n$

as $n\rightarrow\infty \quad \frac{2n+1}{n^2(n+1)^2}\rightarrow\frac{2n}{n^4}=\frac{2}{n^3}$

$$\upsilon_a=\frac{me^4}{4\pi\hbar^3}\cdot\frac{2}{n^3}=\frac{me^4}{2\pi n^3\hbar^3}$$

Hence for $n\rightarrow\infty$

$\upsilon_{classical}\rightarrow\upsilon_Q$ dh

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